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# Using the Plasticity Theory with Taking into Account Rate Sensitivity to Radial Reduction of Porous Bodies

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**ABSTRACT**. For an estimate of force and energy parameters of radial reduction of porous bodies the scalar defining equations of porous body yielding are used with taking into account the rate sensitivity. The expressions for determination of axial and radial stresses obtained. The estimation of hard phase deformation carried out. The constraint equation of the initial and final dimensions with porosity of body obtained. It is shown, that for compaction and obtaining of the given dimensions of porous body the axial stress should exceed radial. The value of side pressure that depends from rheological properties of hard phase, contact area between porous body and instrument and strain rate, estimated. The deformation at static loading leads to a side pressure increase, and dynamic loading reduces it.

**INTRODUCTION.** The radial reduction of porous bodies inside a mould is one of the most spread schemes of deformation. The realisation of this method is possible, when the pressure to a lateral surface is applied through a transmitting medium - glycerin, different liquid oils, melted glass (fig. 1).

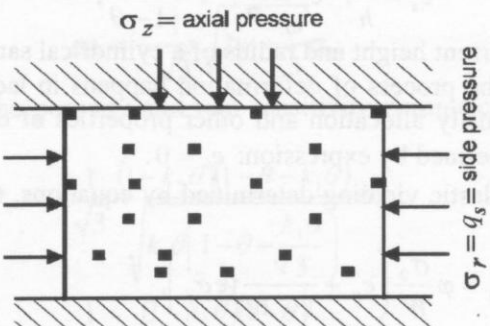


Fig.1. Radial reduction inside a mould with smooth rigid walls

For the estimation of force and energy parameters of radial reduction of porous bodies inside a mould the scalar defining equations of a porous medium plastic yielding with taking into account the strain rate sensitivity were used [1].

The equation of loading surface:

$$\frac{p^2}{\dfrac{1+\nu}{3(1-2\nu)}\varphi}+\frac{\tau^2}{\varphi}=(1-\theta)\sigma_0^2; \tag{1}$$

the equivalent strain rate:

$$W=\frac{\partial\omega}{\partial t}=\frac{1}{\sqrt{1-\theta}}\sqrt{\varphi\gamma^2+\frac{1}{3}\varphi\frac{1+\nu}{1-2\nu}e^2}; \tag{2}$$

the porosity functions with taking into account a strain rate sensitivity:

$$\varphi=\frac{(1-k_2\theta)^2}{1-(1+\frac{1}{3}k_1)\theta}(1-\theta),\qquad \psi=\frac{1}{2}\frac{(1-\theta)(1-k_2\theta)^2}{k_1\theta}; \tag{3}$$

the plastic Poisson's ratio:

$$\nu=\frac{1}{2}\left(1-k_1\frac{\theta}{1-\theta}\right). \tag{4}$$

where $p$ – is the hydrostatic pressure;

$\tau$ – is the intensity of shear stresses;

$\sigma_0$ – is the yielding stress of hard phase;

$\theta$ – is the porosity;

$\varphi$, $\psi$ – are porosity functions with taking into account the rate sensitivity;

$k_1$ – is the coefficient that describing the intensity of densification process;

$k_2$ – is the coefficient that describes the intensity of hardening process of hard phase;

$\gamma$, $e$ – are shape and volume changing rate respectively;

$\omega$ – is the equivalent deformation of porous body.

Generally, at biaxial deformations the parameters in expressions (1-4) are defined through components of stress tensor and strain rate tensor as follows [2]:

$$\gamma=\sqrt{\frac{2}{3}}\left|e_z-e_r\right|,\quad p=\frac{1}{3}\left(\sigma_z+2\sigma_r\right),\quad \tau=\sqrt{\frac{2}{3}}\left|\sigma_r-\sigma_z\right|, \tag{5}$$

thus for a cylindrical sample:

$$e_z=\frac{\dot h}{h},\quad e_r=\frac{\dot R}{R},\quad e=\frac{\dot\theta}{1-\theta}, \tag{6}$$

where $h$ and $R$ - are current height and radius of a cylindrical sample respectively.

We are suppose, that the process of deformation happens in lack of exterior friction. It ensures homogeneity of density allocation and other properties of deformable porous body. The presence of rigid wall defined by expression: $e_r=0$.

According to [2], the plastic yielding determined by equations, that coupling strain rates and stresses as follows:

$$\sigma_{ij}=\varphi\frac{\sigma_0}{W}\left(e_{ij}+\frac{\nu}{1-2\nu}\nu e\delta_{ij}\right), \tag{7}$$

$$e_{ij}=\frac{1}{\varphi}\frac{W}{\sigma_0}\left(\sigma_{ij}-3\frac{\nu}{1+\nu}p\delta_{ij}\right) \tag{8}$$

At biaxial deformations, the equations (7), (8) with taking into account (5) become:

$$e_r=\frac{1}{3\psi(1-2\nu)}\frac{W}{\sigma_0}\left((1-\nu)\sigma_r-\nu\sigma_z\right), \tag{9}$$

$$e_z=\frac{1}{3\psi(1-2\nu)}\frac{W}{\sigma_0}\left(\sigma_z-2\nu\sigma_z\right), \tag{10}$$

and having simple dependencies taking into account the material sensitivity:

$$\sigma_r = \frac{\varphi}{1-2\nu}\frac{\sigma_0}{W}(e_r - \nu e_z), \quad (11)$$

$$\sigma_z = \frac{\varphi}{1-2\nu}\frac{\sigma_0}{W}((1-\nu)e_z - 2\nu e_r). \quad (12)$$

Solving the equations (1-4) together with (5) allows to write expressions for determination of the yield stress for a hard phase $\sigma_0$ and for the equivalent strain rate $W$ in the form:

$$\sigma_0 = \frac{1}{\sqrt{\varphi(1-2\nu)}}\sqrt{\sigma_z^2 - 4\nu\sigma_r\sigma_z + 2(1-\nu)\sigma_r^2}, \quad (13)$$

$$W = \sqrt{\frac{\varphi}{1-2\nu}}\sqrt{(1-\nu)e_z^2 + 4\nu e_r e_z + 2e_r^2}. \quad (14)$$

The further problem is determination of pressure necessary to obtain a given density and dimensions of initial billet.

**DETERMINATION OF PRESSURE DURING DENSIFICATION OF POROUS BODY.** In case of radial pressing $e_z = 0$, therefore from expression (10) follows, that:

$$\sigma_z = 2\nu\sigma_r. \quad (15)$$

Solving the set of equations (12) and (15) becomes to following expressions for axial $\sigma_z$ and radial $\sigma_r$ stresses:

$$\sigma_z = -\sqrt{\varphi}\frac{2}{\sqrt{3}}\frac{\nu}{\sqrt{1-2\nu}}\sigma_{0z}, \quad (16)$$

$$\sigma_r = -\sqrt{\varphi}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{1-2\nu}}\sigma_{0z}, \quad (17)$$

or in variables $\varphi$ and $\psi$ [3]:

$$\sigma_z = -\frac{2}{\sqrt{3}}\frac{\psi - \frac{\varphi}{3}}{\sqrt{2\psi + \frac{\varphi}{3}}}\sigma_{0z}, \quad (18)$$

$$\sigma_r = -\frac{1}{\sqrt{3}}\sqrt{2\psi + \frac{\varphi}{3}}\sigma_{0z}. \quad (19)$$

Expressed through the porosity and rate sensitivity parameters, these components look like:

$$\sigma_z = -\frac{1}{\sqrt{3}}\frac{(1-k_2\theta)(1-\theta-k_1\theta)}{\sqrt{k_1\theta\left(1-\theta-\frac{k_1\theta}{3}\right)}}\sigma_{0z}, \quad (20)$$

$$\sigma_r = -\frac{1}{\sqrt{3}}\frac{(1-k_2\theta)(1-\theta)}{\sqrt{k_1\theta\left(1-\theta-\frac{k_1\theta}{3}\right)}}\sigma_{0z}. \quad (21)$$

We may determine a strain rate of hard phase by using the expression (14), where it is necessary to define $e_z = 0$:

$$W = \sqrt{\frac{\varphi}{1-2\nu}}\sqrt{2}|e_r|, \quad (22)$$

and having introduced porosity functions with taking into account a rate sensitivity [1]:

$$W=\frac{(1-\theta)(1-k_2\theta)}{\sqrt{k_1\theta\left(1-\theta-\frac{k_1\theta}{3}\right)}}\sqrt{2}|e_r|. \tag{23}$$

Based on the continuity equation we shall obtain:

$$e=2e_r,\ e_r=\frac{1}{2}\frac{1}{1-\theta}\frac{d\theta}{dt}. \tag{24}$$

The equation of accumulated deformation of hard phase may be written as:

$$\omega=\frac{1}{\sqrt{2}}\int_{\theta_0}^{\theta}\frac{(1-k_2\theta)}{\sqrt{k_1\theta\left(1-\theta-\frac{k_1\theta}{3}\right)}}d\theta. \tag{25}$$

The solution of this integral is following expression:

$$\omega=\frac{1}{\sqrt{2}}\left[\frac{\sqrt{3\theta}\left(A+\frac{(6+2k_1-3k_2)}{(3+k_1)^{3/2}}B\right)}{C}+\frac{\sqrt{3\theta_0}\left(A_0+\frac{(6+2k_1-3k_2)}{(3+k_1)^{3/2}}B_0\right)}{C_0}\right] \tag{26}$$

In expression (26) it is necessary to define:

$$A=\frac{k_2\sqrt{\theta}((3+k_1)\theta-3)}{3+k_1},\quad A_0=\frac{k_2\sqrt{\theta_0}((3+k_1)\theta_0-3)}{3+k_1},$$

$$B=\sqrt{3-(3+k_1)\theta}\,Arc\sin\left[\frac{\sqrt{3+k_1}\sqrt{\theta}}{\sqrt{3}}\right],\quad B_0=\sqrt{3-(3+k_1)\theta_0}\,Arc\sin\left[\frac{\sqrt{3+k_1}\sqrt{\theta_0}}{\sqrt{3}}\right],$$

$$\sqrt{-k_1\theta((3+k_1)\theta-3)},\quad C_0=\sqrt{-k_1\theta_0((3+k_1)\theta_0-3)}.$$

mpression in conditions of longitudinal strain limitation by rigid dies, the constraint equation of the initial and final dimensions with a porosity of billet exists [4]:

$$S=S_0\frac{(1-\theta)}{(1-\theta_0)}\frac{h}{h_0}. \tag{27}$$

Taking into account the law of porosity change as $\theta=\theta_0e^{-k_1\varepsilon_z}$, the expression (27) is possible to write in the form of:

$$S_0=S\frac{\left(1-\frac{\theta}{e^{-k_1\varepsilon_z}}\right)}{(1-\theta)}\frac{h_0}{h}. \tag{28}$$

The compaction pressure in this case is $\sigma_r$, i.e. $q_s$ related to the area of its application. However, it is necessary to take into account, that for densification and obtaining of necessary dimensions of porous billet, the following condition is necessary $\sigma_z>\sigma_r$, i.e. the response of rigid dies should exceed the applied pressure.

Considering the law of hardening in the Ludwick's form [5] and expression (20) for side pressure gives:

$$q_s=-\frac{1}{\sqrt{3}}\frac{(1-k_2\theta)(1-\theta)}{\sqrt{k_1\theta\left(1-\theta-\frac{k_1\theta}{3}\right)}}(\sigma_{0T}+N\omega^n)S_s, \tag{29}$$

where $S_s$ - is the pressure application area.

**CALCULATION RESULTS.** For given parameters of a detail depending on a loading mode (static or dynamic) the initial dimensions and porosity of billet will be different. The initial porosity is calculating depends on degree of deformation that billet undergoes during deformation. The application of high strain rates allows to use billets with greater initial porosity. When strain rate is growth, in case of other things being equal, a porosity of initial billet increases that contracts force and energy expenditures on its manufacturing. The application of dynamic deformation allows to use billets with greater porosity at constant degree of deformation or smaller degree of deformation at fixed value of initial porosity.

In the fig. 2 the dependence of relative side pressure on porosity presented at radial compression inside a mould with smooth rigid walls. Besides of rheological properties of a hard phase and pressure application area, the value of side pressure depends on implemented strain rate. The static deformation raises a common value of a side pressure, and dynamic reduces it.

The expression (15) and data on influence of strain rate on plastic Poisson's ratio [1] allow to estimate the value of axial pressure. At the fixed value of side pressure the necessary axial pressure, as the response of rigid dies, that are limiting the yielding of porous billet in axial direction, will be so much lower, than strain rate greater. When the initial porosity of billet growth this pressure is also diminished.

The expressions for calculation of the initial dimensions of billet (28) and compaction pressure (29) can be used for solving of different problems of powder materials deformation by defining in equation (29) the rheology of hard phase, i.e. mechanism of its yielding.

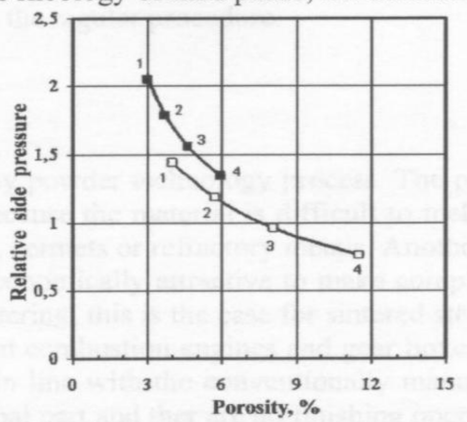


Fig. 2. Dependence of relative side pressure on porosity at radial compression inside a mould with smooth rigid walls: □ – dynamic deformation; ■ – static deformation, 1 - $\varepsilon_z = 0.20$; 2 - $\varepsilon_z = 0.30$; 3 - $\varepsilon_z = 0.40$; 4 - $\varepsilon_z = 0.50$

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