Materials Science Forum Vols. 558-559 (2007) pp. 563-567
online at http://www.scientific.net

Modelling of the dynamic processes of structure formation by macroscopic parameters of plastic deformation

Lyudmila Ryabicheva, Dmytro Usatyuk

Dept. of Material Science, East-Ukrainian Volodymir Dal National University

Molodiozhny block, 20A, Lugansk, Ukraine, 91034

e-mail: ryabic@cci.lg.ua

Keywords: dynamic recrystallization, recrystallized volume, growth rate, oscillating function, grain growth intensity coefficient.

Abstract. In this paper, the phenomenological approach which connects the recrystallization parameters with thermomechanical characteristics of deformation is implemented to simulation of structural transformations. Two cases were considered, when the growth rate of recrystallized volumes is damping fast, and when a weak growth of recrystallized grains is observed. A pattern of structural states was obtained among the models with fast decrease in growth rate of recrystallized grain and weak grain growth. The abatement of the sensibility to the intensity of grain growth with increasing of strain rate was established. The function of recrystallized volume has oscillating behaviour depending on time and coefficient of recrystallization intensity. These results were confirmed by experimental researches of rheological behaviour of steels at low strain rates and high temperatures.

Introduction

The development of models for processes of dynamic and static polygonization and recrystallization are based on two approaches: phenomenological that allows to connect the recrystallization parameters with thermomechanical characteristics of deformation and structural allowing to define the necessary thermomechanical characteristics of deformation on the basis of simulation of cooperative dislocation movement processes and appearance of grain embryos. Realization of both approaches is complicated metallophysically [1] as well as mathematically [2]. Nevertheless the necessity of solving such a problem is determined by requirements to real manufacturing process. The phenomenological approach of problem solving demands application of multidimensional space of structural parameters (mesh size, dislocation density, concentration of carbides) that depend on thermomechanical characteristics of deformation (temperature, invariants of deformation tensor, strain rate, stresses). At the same time the initial physicochemical constants of structure formation are not taking into account. Such approach is efficient only in case of large number of experimental data in investigation of plastic deformation at different temperatures and strain rates. In this paper, the phenomenological approach is presented for estimation of dynamic recrystallization parameters depend on growth speed of recrystallization volumes.

Mathematical Model

Two cases of recrystallized volumes growth at the specified thermomechanical characteristics of deformation will be considered: the first, when a growth rate of recrystallized volumes is damping fast [2], and the second, when a weak growth of recrystallized grains is observed [3].

The Case of Fast Damping of Recrystallized Volumes Growth Rate. In this case we should use a differential equation describing change of recrystallized volume at time depend on recrystallization intensity $\alpha = \alpha(\theta)$ at different temperatures θ and strain rates $\dot{\varepsilon}$:

$$\frac{dV_n(\tau)}{d\tau} = \gamma[V_n(\tau-1) - V_n(\tau)], \quad \tau \in (\tau, +\infty), \tag{1}$$

where $V_n(\tau)$ - is the recrystallized volume depends on time τ;

γ - is the adjusted coefficient of recrystallization intensity defined by the following ratio $\gamma = \alpha(\theta)/\nu(\dot{\varepsilon})$, ($\nu$ - is the structure change speed at the strain rate $\dot{\varepsilon}$).

We use the following boundary conditions for solving the Eq. 1:

$$V_n(\tau) = 0 \ \ npu \ \ \tau \in (0, 1), \ V_n(1) = 1. \tag{2}$$

Then solve the Eq. 1 by operational method. The desired quantity $V_n(\tau)$ discontinues at the point $\tau_1 = 1$. In the operational calculus differentiation of original function presumed that it is continuous at $\tau > 0$, but it doesn't contains into the solution. Therefore we need to use the following operational ratio for the original function $U(\tau)$ with points of discontinuity $\{\tau_i\}\ i = 1$,
$0 = \tau_0 < \tau_1 < \tau_2 < \ldots$, where $C_i = \lim\limits_{\tau \to \tau_i^+} U(\tau) - \lim\limits_{\tau \to \tau_i^-} U(\tau)$, are final,

$$\text{moreover: } \sum_{i=1}^{\infty} |C_i| < \infty, \ U(\tau) = pU(p) - U(0) - \sum_{i=1}^{\infty} C_i e^{-p} i. \tag{3}$$

Here $U(p)$ - is the Laplace transform of discontinuous original function $U(\tau)$.

Let $V_n(\tau) = V(p)$, then $V_n^{`}(\tau) = pV(p) - V(0) - 1e^{-p} = pV(p) - e^{-p}$,
because of $\tau_1 = 1$ and $V(0) = 0$, so $C_i = \lim\limits_{\tau \to \tau_i^+} U(\tau) - \lim\limits_{\tau \to \tau_i^-} U(\tau) = 1$.

According to the shift theorem $V_n(\tau - 1) = e^{-p} V(p)$.

As a result the Eq. 1 will transform to the operational equation:

$$pV(p) - e^{-p} = \gamma\left[e^{-p} V(p) - V(p)\right]. \tag{4}$$

After the transformation it looks like:

$$V(p) = e^{-p} \frac{1}{p\left[1 - \frac{\gamma}{p}\left(e^{-p} - 1\right)\right]} = e^{-p} \sum_{i=0}^{\infty} \frac{\gamma^i}{p^{i+1}} \left(e^{-p} - 1\right)^i = e^{-p} \sum_{i=0}^{i} \frac{\gamma^i}{p^{i+1}} \sum_{q=0}^{i} (-1)^{i-q} C_1^q e^{-pq} =$$

$$= e^{-p} \sum_{q=0}^{\infty} \frac{\gamma^q}{q!} \sum_{i=q}^{\infty} (-1)^{1-q} \frac{\gamma^{i-q}}{p^{i-q+i+q}} A_i^q e^{-pq} = e^{-p} \sum_{q=0}^{\infty} \frac{\gamma^q}{q!} \sum_{i=q}^{\infty} (-1)^{i=q} i(i-1)\ldots(i-q-1) \frac{\gamma^{i-q}}{p^{i-q+i+q}} e^{-pq} = \tag{5}$$

$$= e^{-p} \sum_{q=0}^{\infty} \frac{\gamma^q}{q!} (-1)^q F^{(q)}(p) e^{-pq},$$

where $F(p) = \frac{1}{p} - \frac{\gamma}{p^2} - \frac{\gamma^2}{p^3} - \ldots = \sum\limits_{k=0}^{\infty} (-1)^k \frac{\gamma^k}{p^{k+1}} = \sum \frac{(-\gamma\tau)^k}{k!} = e^{-\gamma\tau} = f(\tau)$.

According to the image differencing attribute:

$F_{(p)}^{(q)} = (-1)^q \tau^q f(\tau)$

It is given by:

$$V(p)\sum_{q=0}^{\infty}\frac{\gamma^q}{q!}F_{(p)}^{(q)}e^{-p(q+1)}(-1)^{-q} = \sum_{q=0}^{\infty}\frac{\gamma^q}{q!}(\tau-q-1)^q e^{-\gamma(\tau-q-1)}\eta(\tau-q-1) = V_n(\tau), \tag{6}$$

where $t \in (0, +\infty)$.

The dynamics of a recrystallized volume $V_n(\tau)$ is described by the oscillating function. In addition, the adjusted coefficient of recrystallization intensity γ defines damping of oscillations (Fig. 1).

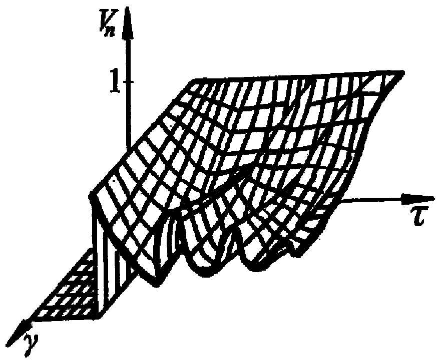

Fig. 1. The dependence of the recrystallized volume on time and adjusted coefficient of recrystallization intensity [2]

It is significant, that recrystallized volume at $\tau \to \infty$ is:

$$V_n(\infty) = \lim_{p\to 0} pV(p) = \frac{1}{1+\gamma} \tag{7}$$

The Case when a weak growth of recrystallized grains is observed.

Under these conditions the model of a recrystallized volume change $V_n(\tau)$ at the time may be expressed in the following way:

$$\frac{dV_n(\tau)}{d\tau} = \frac{\gamma}{\nu}\left[L(1)V_n(\tau-1) - \int_{\tau-1}^{\tau} V_n(\xi)G(\tau-\xi)d\xi\right], \ \tau \in (1, \ +\infty), \tag{8}$$

with following boundary conditions: $V_n(1) = 1;\ V_n(\tau) = 0;\ \tau \in (0,\ 1)$,

where $L(1)$ - is the maximal volume of grain grown during the time unit of structure change;
$G(\tau-\xi)$ - is the grain growth speed into the structural state ξ of multidimensional space that depends on structure change rate ν at the moment of deformation τ.

It is necessary to use the Laplace transform for solving the Eq. 8. Let us assume that:

$G(\tau) = \beta\tau,\ \beta = \beta(\theta)$ - is the grain growth intensity coefficient.

It is given by:

$$\int_{\tau-1}^{\tau} V_n(\xi)G(\tau-\xi)d\xi = \beta\left[\int_0^{\tau} V_n(\xi)G(\tau-\xi)d\xi - \int_0^{\tau} V_n(\xi)G(\tau-1-\xi)d\xi - \int_0^{\tau} V_n(\xi)d\xi\right] =$$
$$= \beta\left[V(p)\frac{1}{p^2}\left(1-e^{-p}\right) - e^{-p}\frac{V(p)}{p}\right].$$

Taking into account, that $L(1) = \int_0^1 G(\tau)d\tau = \beta\int_0^1 \tau d\tau = \frac{\beta}{2}$, we are coming to the operational equation:

$$pV - e^{-p} = \frac{\gamma}{\nu}\left[\frac{\beta}{2}Ve^{-p} - \beta V\frac{1}{p^2}\left(1-e^{-p}\right) + \beta\frac{V}{p}e^{-p}\right].$$

From this it follows that:

$$V(p) = \frac{e^{-p}}{p - \frac{\gamma\beta}{2\nu}e^{-p} + \frac{\gamma\beta}{\nu}\left[\left[\frac{\left(1-e^{-p}\right)}{p^2} + \frac{e^{-p}}{p}\right]\right]}. \tag{9}$$

Further it looks like:

$$V_n(\infty) = \lim_{p\to 0} pV(p) =$$
$$= \lim_{p\to 0}\frac{e^{-p}}{1 - \frac{\gamma\beta}{2\nu}\left(\frac{1}{p} - 1 + \frac{p}{2!}\ldots\right) + \frac{\gamma\beta}{\nu}\frac{p - \frac{p^2}{2!} + \frac{p^3}{3!}\ldots + p\left(1 - p + \frac{p^2}{2!} - \frac{p^3}{3!} + \ldots\right)}{p^3}} = \frac{1}{1 + \frac{1}{6}\frac{\gamma\beta}{\nu}} = \frac{1}{1+K\gamma}, \tag{10}$$

where $K = \beta/6\nu$ - is the adjusted coefficient of growth intensity of recrystallized grains.

Let us transform the expression Eq. 10 for conversion to the original function $V_n(\tau)$ and further it looks like:

$$V(p) = \sum_{i=0}^{\infty}\left(\frac{\gamma\beta}{\nu}\right)^i_{q_1+q_2+q_3+q_4=i}\frac{i!}{q_1!q_2!q_3!q_4!}\left(\frac{1}{2}\right)^{q_1}(-1)^{q_4}\frac{1}{p^{q_2+2q_3+2q_4+i+1}}e^{-(1+q_1+q_2+q_3)p}.$$

From this it follows that:

$$V_n(\tau) = \sum_{i=0}^{\infty}\left(\frac{\gamma\beta}{\nu}\right)^i_{q_1+q_2+q_3+q_4=i}\frac{i!}{q_1!q_2!q_3!q_4!}\left(\frac{1}{2}\right)^{q_1}(-1)^{q_4}\frac{\tau^{q_2+2q_3+2q_4+i} - (1+q_1+q_2+q_3)}{(q_2+2q_3+2q_4+i)!} *$$
$$* \eta[\tau - (1+q_1+q_2+q_3+i)].$$

Using the following expression: $\alpha(\theta) = \frac{1-V_n(\infty)}{V_n(\infty)}\nu(\dot{\varepsilon})$, it is possible to create the pattern of structural states – the family of projections of isolevels $V_n(\infty)$ onto the plane α, ν (Fig. 2).

During the numerical investigation of the model Eq. 9 with taking into account grain growth with linear growth function $G(\tau-\xi)=\beta(\tau-\xi)$ was established that behavior of the expression Eq. 7 is qualitatively equivalent to the Eq. 9. However, at the steady state a recrystallized volume is different at the value of $K=\beta(\theta)/Gv(\varepsilon)$, that testify of sensitivity decrease to the grain growth intensity at increasing of strain rate. Depending on the value of K the possible case is reduction or grain growth (Fig. 3). The similar dependences obtained at austenitic steels [4].

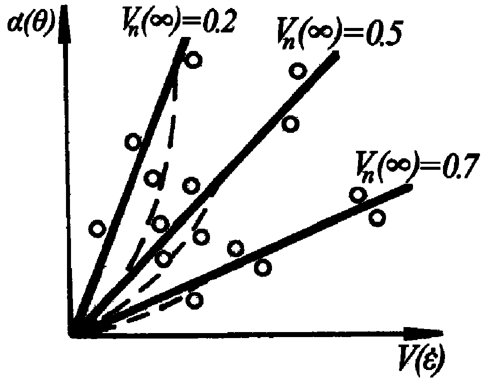

Fig. 2. The patterns of structural states, created according to the following models:
——— - is fast damping of grain growth rate;
- - - - - - is the weak growth of grains;
o - are calculated points [2]

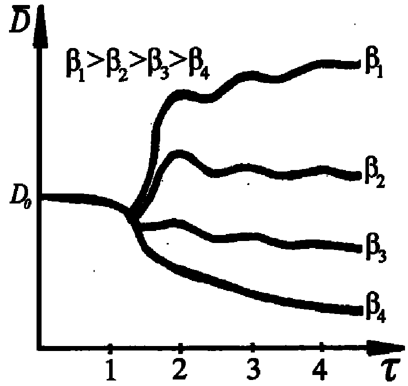

Fig. 3. The kinetics of changing the grain size depends on growth intensity [4]

As a result of solving the Eq. 1 and Eq. 8 obtained that the function $V_n(\tau,\gamma)$ shows oscillating behavior. This statement was confirmed by the researches of rheological behavior of steels at low strain rates and high temperatures, and also with the results of paper [2], when the oscillating behavior of a deformation resistance observed in conditions leading to the same effect in kinetics of recrystallized volume. Solving the Eq. 1 and Eq. 8 may be finally obtained if constants α, β, γ are known. If these constants are known, it is possible to define the value of recrystallized volume as a function of time and strain rate. However the difficulty in solving of this task is concerned with defining of these constants.

Summary

The model with taking into account growth rate of recrystallized grain and possibility of its reduction or growth was created. It is shown clearly, that the function of recrystallized volume shows oscillating behavior in depends on time and the adjusted coefficient of recrystallization intensity.

References

[1] B. Kowalcki, C.M. Sellars, M. Pietrzyk: *Identification of rheological parameters on the basis of plane strain compression tests on specimens of various initial dimensions*, Computational materials science, 35 (2006), p. 92-97.

[2] I.G. Gunn: *Mathematical modelling of structural transformations at hot deformation of metals*, Metals, Vol 5 (1989), p. 82-85.

[3] A.A. Baranov, V.P. Gorbatenko, A.L. Geller: *The ways of combination of deformational-thermal treatment of steel*, Metals and Casting of Ukraine, Vol. 4 (1995), p. 8-13.

[4] S. Sakai, T. Sakai, K. Takeishi: *Effect of strain rate and temperature on the hot-worked* structure of a 0.1%-carbon steel, Tetsu-to-Hagane, Vol. 62 (1976), p. 856.